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RAPIDC: RAPID CORRELATIONS WITH FFT.(U)

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USL Problem No.
A-652-00-00

Navy Underwater Sound Laboratory
New London, Connecticut

MOST Project-3

RAPIDC:
RAPID CORRELATIONS WITH FFT

by

G. Clifford Carter and Charles R. Arnold

NUSL Technical Memorandum No. 2070-6-70

15 January 1970

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INTRODUCTION

MOST Project

The correlation function of two signals can rapidly be obtained by taking the inverse discrete fast Fourier transform (IDFFT) of the product of the discrete fast Fourier transform (DFFT) of the first signal times the complex conjugate of the DFFT of the second signal. The subroutine RAPIDC accomplishes this operation.

In a preliminary report to the NUSL, the Sperry Rand Corporation reports (reference (a)) success in performing correlation by FFT as follows:

The discrete fast Fourier transform (DFFT) of signal 1 and the DFFT of signal 2 are taken. The complex conjugate of the latter is then multiplied term by term, with the former transform. The inverse discrete fast Fourier transform (IDFFT) when properly scaled yields the correlogram of signal 1 cross (or replica) correlated with signal 2.

Continued developments in the field of the discrete fast Fourier transform (reference (b)) make implementation of this concept easier and more frequent (reference (c)). The primary advantage of correlations with FFT is the high rapidity with which the computations can be performed. So staggering is the time element that it is conceivable to disallow time domain replica and cross correlations by the classical method for high order cases.

Appendix A is a program listing with one applicable subroutine, CORRAD, given in Appendix B. The subroutine for FFT can be found in

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reference (b) or the user may prefer to use another transformation subroutine of his own choosing. Appendix C gives a sample case of cyclic and acyclic correlation of two pulses.

THEORY

The time domain definition of the correlation function is well known for real causal functions, $x(t)$ and $y(t + \tau)$ to be:

$$\phi_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t) y(t + \tau) dt \quad (1A)$$

$$\phi_{xy}(k) = \frac{1}{NNN} \sum_{N=1}^{NNN} X(N) Y(N+k) \quad (1B)$$

Equation (1A) from Bendat and Piersol (reference (c)) (digitally expressed as Eq. (1B) in reference (i)) is a classical definition requiring large and often excessive amounts of computation time for large arrays $X(N)$ and $Y(N)$. The FORTRAN implementation of this definition for the discrete case is straight forward and its use throughout government and industry is quite common. The basic definition is sometimes modified and a normalized version forces ϕ_{xy} to fall between +1 and -1.

It has long been known that the inverse transform of the cross spectral density function is the correlation function. Davenport and Root (reference (d)) state that:

$$\phi_{xy}(\tau) = \int_{-\infty}^{\infty} S_{xy}(f) e^{j\omega\tau} df \quad (2)$$

$$\omega = 2\pi f$$

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where ϕ_{xy} is the correlation function and S_{xy} is the cross spectral density function.

The cross spectral density function is, in general, complex consisting of a real part, the co (coincidental) spectral density function, and an imaginary part, the quadrature spectral density function (reference (c)).

The cross spectral density function $S_{xy}(\omega)$ of $x(t)$ and $y(t)$ referred to by Papoulis (reference (f)) as the cross spectrum is given by:

$$S_{xy}(\omega) = X(\omega) \cdot Y^*(\omega) \quad (3)$$

where '*' indicates conjugation

$$x(t) \longleftrightarrow X(\omega)$$

$$y(t) \longleftrightarrow Y(\omega)$$

and \longleftrightarrow indicates transform pair.

Recently, Singleton (reference (b)) has presented an FFT subroutine coded in FORTRAN which can be called as follows:

CALL FFT(XX,YY,NNN,NNN,NNN,ISN)

where $(XX + jYY)$ is a complex input and NNN is the Number of Values to be transformed (NNN must contain no prime factor greater than 23) and where for $ISN = -1$ the forward discrete fast Fourier transform is computed and returned as the complex output $(XX + jYY)$. Similarly for $ISN = 1$ the inverse discrete fast Fourier transform is computed and returned as the complex output (NNN times $(XX + jYY)$).

Given the two real signals $x(t)$ and $y(t)$ they can be transformed as the complex signal $(x(t) + jy(t))$ yielding the complex transform, $Z(\omega)$, of the complex input signal. The transform of $x(t)$ and $y(t)$ can be recovered as shown by Cooley (reference (h)) as follows:¹

$$X(\omega) = (Z(\omega) + Z^*(-\omega))/2 \quad (4)$$

$$Y(\omega) = (Z(\omega) - Z^*(-\omega))/2j \quad (5)$$

¹ A brief and complete derivation of Eq. (4) and (5) is shown by Eby in reference (g).

Multiplication term by term of $X^*(\omega)$ with $Y(\omega)$ can be shown from Eqs. (4) and (5) to yield the real (CO) and imaginary (QUAD) parts of the cross spectral density function, $S_{xy}(\omega)$, as follows:

$$CO(\omega) = 0.50 (XX(\omega)YY(-\omega) + XX(-\omega)YY(\omega)) \quad (6)$$

$$jQUAD(\omega) = 0.25 (XX(\omega)^2 + YY(\omega)^2 - XX(-\omega)^2 - YY(-\omega)^2) \quad (7)$$

where $XX(\omega) + jYY(\omega)$ is the complex transform output ($Z(\omega)$) of Singleton's FFT subroutine.

Implementation of these two equations in FORTRAN with positive indexing and taking advantage of symmetries present is done by the Subroutine COQUAD. A listing of COQUAD is enclosed.

Taking the inverse discrete Fourier transform of the CO and Quadrature spectral density functions, properly scaling the output gives the correlation function equivalent to Eq. (1).

Removal of the mean and division by the standard deviation of the data prior to the described procedure for RAPID Correlation yields a normalized correlation coefficient between plus and minus one.

SUBROUTINE RAPIDC

Subroutine RAPIDC is called from the main program as follows:

CALL RAPIDC(XX,YY,NUMX,NUMY,N2PLOT,NOWRAP)

where XX is an array (dimensioned at least NUMY for NOWRAP = 1 and dimensioned at least NUMX plus NUMY for NOWRAP = 0)

YY is an array larger than or equal to XX (dimensioned in the main program with the rules above for the XX array)

NUMX is the number of values in XX

NUMY is the number of values in YY

N2PLOT is the number to plot and is supplied by the subroutine

NOWRAP is the flag parameter for cyclic and acyclic correlation; NOWRAP = 0 gives acyclic correlation (i.e., zero wrap around) but requires more array storage. See discussion which follows.

CYCLIC AND ACYCLIC OPTION

In the call to RAPIDC, if NOWRAP = 0 then acyclic correlation is performed. However, in the main program both of the arrays must be dimensioned to at least NUMX + NUMY. For cross correlation of arrays with 8000 values both arrays would be dimensioned to 16000.

If NOWRAP = 1 then cyclic correlation is performed and a wrap around effect takes place. If a replica correlation is taking place the user should note that no wrap around will occur until NUMY - NUMX correlations take place. The value of NUMY - NUMX will be returned as the number to plot in cases where NOWRAP = 1. An example would be of the replica correlation of 1000 values with 16000. No wrap around would take place until 15000 correlations for NOWRAP equal to 1 and the N2PLOT would equal 15000.

The authors suggest as a rule of thumb that NOWRAP be set equal to 0 for cross correlation with both data arrays 'over' or 'double' dimensioned and that NOWRAP be set equal to 1 replica correlations with the user only plotting N2PLOT number of values supplied by the subroutine.

LIMITATIONS OF THE FFT

Limitations of Singleton's FFT are that it must be called with a highly composite number (that is one with no prime factor greater than 23). The subroutine RAPIDC calls FFT with NUMY for NOWRAP=1 and calls FFT with NUMX + NUMY for NOWRAP = 0. A table of highly composite numbers is available in reference (b).

TIMING CONSIDERATIONS

Run and timed on the UNIVAC 1108 under EXEC II level 22 compiler Replica Correlation of 1000 values with 15000 values took 13 seconds with RAPIDC. The exact same case processed by the discrete version of the classical Eq. (1) required 182 seconds. Timing included no input-output manipulations. This increase in speed in the ratio of 14:1 is significant and should not be overlooked when considering correlation computations.

LOCATION OF RAPIDC

RAPIDC, FFT and COQUAD are located on File one of CUR Tape U241. Both the symbolic and relocatable versions are stored on U241, so these three subroutines may be used, compiled, listed or punched from tape. The authors request that the tape be brought into the PCF as follows:

▽R ASG A = U241
▽ XQT CUR
IN A
TRI A

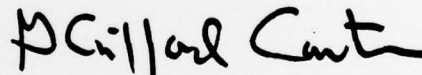
The tape U241 should be listed on a yellow run request card as an input tape (left column) and circled.

AN I/O PACKAGE FOR RAPIDC

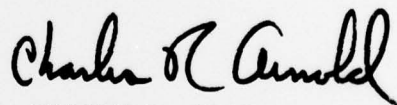
An input-output package, computer program S1612, has been written and is stored on File 1 of CUR Tape U241. S1612 is a general-purpose FFT replica correlation program with basically the same input and output as S1530 (reference (1)). The primary difference between S1530 and S1612 is the subroutine which performs the correlation. S1612 uses RAPIDC while S1530 used a classical time domain approach. Other coding differences are required because RAPIDC uses an 'in place' algorithm. Still, run time has been reduced tenfold with S1612 over the previous time domain method.

ACKNOWLEDGEMENTS

The assistance of J. P. Burke, F. S. White and J. F. Ferrie was greatly appreciated in development of this digital signal processing subroutine.



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LIST OF REFERENCES

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- (e) T. H. Glisson, C. I. Black and A. P. Sage, On Digital Replica Correlation with Applications to Active Sonar, IEEE Transactions on Audio and Electroacoustics, Vol. AU-17, No. 3, September 1969.
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- (g) E. S. Eby, "Simultaneous Computation of the Fourier Transforms of Two Real Functions," USL Technical Memorandum No. 2242-262-67.
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- (i) C. R. Arnold, G. C. Carter and J. F. Ferrie, "A Replica Correlation Program," USL Technical Memorandum No. 2070-406-69, 5 November 1969.

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APPENDIX A

```
C SUBROUTINE RAPIUC ** C.R. ARNOLD AND G.C. CARTER ** 11/03/69
C
C
C
C
C SUBROUTINE RAPIUC(XX,YY,NUMX,NUMY,NZPLOT,NOWRAP)
C
C
C
C COMMENT - RAPIUC IS A RAPID CORRELATION SUBROUTINE BY FFT
C
C
C
C COMMENT - RAPIUC CROSS CORRELATES XX WITH YY
C COMMENT - NUMY IS THE NUMBER OF VALUES IN THE YY ARRAY
C COMMENT - NUMX IS THE NUMBER OF VALUES IN THE XX ARRAY
C COMMENT - THE XX ARRAY IS SHORTER OR EQUAL TO THE YY ARRAY
C COMMENT - NUMX AND NUMA+NUMY MUST BE HIGHLY COMPOSITE
C (THAT IS -- NO PRIME FACTOR GREATER THAN 23)
C COMMENT - NZPLOT IS THE NUMBER OF VALUES TO BE PLOTTED BY THE USER
C COMMENT - NZPLOT IS SUPPLIED TO THE USER BY THE SUBROUTINE
C COMMENT - NOWRAP IS A FLAG PARAMETER FOR CYCLIC AND ACYCLIC CORRELATION
C (THAT IS -- THERE IS NO WRAP AROUND IF NOWRAP=0 --HOWEVER
C MORE STORAGE IS REQUIRED TO SUPPRESS WRAP AROUND)
C COMMENT - AFTER PROCESSING XX(I) HOLDS THE CORRELOGRAM
C COMMENT - FOR NOWRAP=1, BOTH XX AND YY MUST BE DIMENSIONED TO
C NUMY IN THE MAIN PROGRAM
C COMMENT - FOR NOWRAP=0, BOTH XX AND YY MUST BE DIMENSIONED TO
C NUMX + NUMY IN THE MAIN PROGRAM
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```
C
C
C      CONTINUE
C
C      600 CONTINUE
C
C      COMPUTE FAST FOURIER TRANSFORM BY SINGLETON'S METHOD
C
C      ISN=-1
C      CALL FFT(XA,YY,NEWNNN,NEWNNN,NEWNNN,ISN)
C
C      COMPUTE COINCIDENTAL AND QUADRATURE SPECTRA
C
C      CALL COQUAD(XX,YY,NEWNNN)
C
C      COMPUTE INVERSE FAST FOURIER TRANSFORM BY SINGLETON'S METHOD
C
C      ISN=1
C      CALL FFT(XA,YY,NEWNNN,NEWNNN,NEWNNN,ISN)
C
C      SCALE INVERSE FFT OUTPUT
C
C      FLNUMA=FLOAT(NUMA)
C      FLNEWN=FLOAT(NEWNNN)
C      FSCALE=FLNUMA*FLNEWN
C      IF(NOWRAP)/10,730,710
C      710 DO 720 I=1,NUMY
C          XX(I)=XX(I)/FSCALE
C      720 CONTINUE
C          GO TO 750
C      730 DO 740 I=1,NUMXPY
C          XX(I)=XX(I)/FSCALE
C      740 CONTINUE
C      750 CONTINUE
C
C      RETURN TO MAIN PROGRAM WITH CORRELOGRAM IN XX
C
C      RETURN
C      END
```

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APPENDIX B

```
C      SUBROUTINE COQUAD **C.R.ARNOLD ** UPDATED 11/03/69
C
C      SUBROUTINE COQUAD (XX,YY,NEWNNN)
C
C      DIMENSION XX(1),YY(1)
C
C      NNP2=NEWNNN+2
C      IHNPI=(NEWNNN/2)+1
C
C      XX(1)=XX(1)*YY(1)
C      YY(1)=0.0
C
C      DO 675 K=2,IHNPI
C      J=NNP2-K
C      XX=0.50*(XX(K)*YY(J)+XX(J)*YY(K))
C      YY(K)=0.25*(XX(J)**2+YY(J)**2-XX(K)**2-YY(K)**2)
C      YY(J)=-YY(K)
C      XX(K)=XXX
C      XX(J)=XX(K)
C 675 CONTINUE
C
C      RETURN
C      END
```

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